Math 175: 3-D coordinate systems (13.1)

(1) To locate a point in space, two coordinates x and y is not enough. By introducing an extra coordinate z we can tell where is exactly the point. Thus we have a three dimensional coordinate system. Similar to two dimensional system, we have three pairwise perpendicular directed lines (called axis) intersecting at the same point O (the origin). The three lines separate the space into eight octants, and six coordinate planes.

(2) For each point P, we have coordinate (x, y, z). We can project this point to the coordinate planes, called the projection of P.

In 3-D system, an equation involving x, y and z represents a surface in $R^3 = \{(x, y, z) : x, y, z \in R\}$.

Examples 1: What surfaces in R^3 are represented by the following equations: (a) $z = 3$; (b) $y = 5$.

Examples 2: Describe and stretch the surface in R^3 represented by the equation $y = x$.

(3) As in R^2, there is a formula to find the distance between two points $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ in R^3:

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Can you prove it?

Example 3: Find an equation of a sphere with radius r and center (h, k, l).

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2.$$

Example 4: Show that $x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$ is the equation of a sphere.

Example 5: What region in R^3 is represented by the following inequalities?

$$1 \le x^2 + y^2 + z^2 \le 4, \qquad z \le 0.$$