

Math 274 Homework Seven

Choose to work out at least four problems from the following five problems, including the first one. Due: Tuesday, March 18, 2008

- (1) Let $p_e(n)$ and $p_o(n)$ denote the number of partitions of n into an even number and an odd number of distinct parts, respectively. Show that $p_e(n)$ equals $p_o(n)$ except when n has the form $\sum_{i=1}^m (m+i)$ or $\sum_{i=0}^{m-1} m+i$, in which case they differ by one. (See hint in 3.4.21 in the textbook)
- (2) Let a_n be the number of 3-part partitions of n with largest part less than $n/2$. For $n \geq 3$, prove that

$$a_n = a_{n-3} + \begin{cases} 0 & \text{if } n \text{ is even} \\ \lfloor \frac{n+1}{4} \rfloor & \text{if } n \text{ is odd.} \end{cases}$$

- (3) Find the number of integers between 1 and 10000 that are not divisible by 4, 6, 7, or 10.
- (4) Determine the number of integral solutions of the equation $x_1 + x_2 + x_3 + x_4 = 20$ that satisfy

$$1 \leq x_1 \leq 6, 0 \leq x_2 \leq 7, 4 \leq x_3 \leq 8, 2 \leq x_4 \leq 6.$$

- (5) Count the permutations $i_1 i_2 i_3 i_4 i_5 i_6$ of $[6]$, where $i_1 \neq 1, 5$; $i_3 \neq 2, 3, 5$; $i_4 \neq 4$ and $i_6 \neq 5, 6$.