Math 274 Homework Six

Choose to work out at least four problems from the following five problems, including the first one. Due: Tuesday, March 11, 2008

(1) Let a_n be the number of words of length n from the alphabet $\{w, x, y, z\}$ such that x appears an even number of times and y appears an odd number of times. Build the EGF for $< a >$ and use it to obtain a formula for a_n.

(2) Evaluate $\sum_{k=0}^{m} \binom{n}{k}\binom{n-k}{m-k}$ using binomial convolution, and give a bijective proof of the resulting identity.

(3) Let a_n be the number of involutions on an n-element set. Derive a recurrence for $< a >$, and use the generating function method to obtain the EGF from it.

(4) Let a_n be the number of permutations of $[n]$ in which every cycles has odd length. Let b_n the number in which every cycle has odd length and there are an even number of cycles. Let $A(x)$ and $B(x)$ denote their EGFs respectively. Use the Exponential Formula to prove that $A(x) = (\frac{1+x}{1-x})^{1/2}$ and $B(x) = (1-x^2)^{-1/2}$.

(5) Let B_n be the total number of partitions of $[n]$.
a) Prove that $B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k$ for $n \geq 0$.
b) Use part a) to prove that the EGF for B_n is $e^{e^x - 1}$.