Q: Find equation of the plane containing the line of intersection of $7x-14y=5$ & $3z-3y-8=0$, and also perpendicular to $2x+4y-3z=0$.

A: Let γ be the plane whose equation we have to find and the other planes be denoted as:

$$\begin{cases} \gamma_1 : 7x-14y=5 \\ \gamma_2 : 3z-3y-8=0 \\ \gamma_3 : 2x+4y-3z=0 \end{cases}$$

Now, γ contains the line (say L) of intersection of γ_1 & γ_2, and γ is perp. to γ_3

To find the equ. of γ, we find the following

(i) <u>a point on γ</u>: It is enough to find a point on L since L lies on γ. Let (p,q,r) be a point on L (= intersection of γ_1 & γ_2).

Hence, (p,q,r) lies on γ_1 & γ_2. Equations of

$$\gamma_1 \ \& \ \gamma_2 \Rightarrow \begin{cases} 7p-14q=5 \\ 3r-3q-8=0 \end{cases} \quad \Bigg| \quad \text{Set } q=0 \Rightarrow \begin{cases} p=\frac{5}{7} \\ r=\frac{8}{3} \end{cases}$$

Thus, $(5/7, 0, 8/3)$ is a point on γ.

(ii) a vector normal to γ: If we can find two vectors $\underline{v}_1$ & $\underline{v}_2$ lying on γ, then $\underline{v}_1 \times \underline{v}_2$ is normal to γ. So, it is enough to find two non-parallel vectors on γ.

Since γ_3 is perp. to γ, normal of γ_3 lies on γ. Let $\underline{v}_1$ = normal of $\gamma_3 = \langle 2, 4, -3 \rangle$

Now, since L lies on γ, any vector along L lies on γ. Again, since L lies on γ_1 & γ_2, therefore L is perpendicular to the vectors $\underline{n}_1$ & $\underline{n}_2$ which are normal to γ_1 & γ_2 respectively. Equivalently, L is perp. to the plane containing $\underline{n}_1$ & $\underline{n}_2$. Hence, $\underline{n}_1 \times \underline{n}_2$ is along L. From equations of γ_1 & γ_2, one can choose $\underline{n}_1 = \langle 7, -14, 0 \rangle$ & $\underline{n}_2 = \langle 0, -3, 3 \rangle$.

$$\underline{n}_1 \times \underline{n}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 7 & -14 & 0 \\ 0 & -3 & 3 \end{vmatrix} = \langle -42, -21, -21 \rangle = -21 \langle 2, 1, 1 \rangle$$

Set $\underline{v}_2 = \langle 2, 1, 1 \rangle$ which lies on L & hence on γ.

So, $$\underline{v}_1 \times \underline{v}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 4 & -3 \\ 2 & 1 & 1 \end{vmatrix} = \langle 7, -8, -6 \rangle = \text{normal to } \gamma.$$

Equation of γ is :

$$7\left(x - \tfrac{5}{7}\right) - 8\left(y - 0\right) - 6\left(z - \tfrac{8}{3}\right) = 0$$