Homework 1
Geology 390

1) Plot the following function $z(x)$ for: (a) $A = 1$, $\lambda = 2$; (b) $A = 0.5$, $\lambda = 6$. Plot both cases (a) and (b) on the same graph over $x = 0$ to $x = 6$.

$$z(x) = A\cos\left(\frac{2\pi}{\lambda}x\right)$$

2) Plot the following function $z(x)$ for: $A = 1$, $\lambda = 2$.

$$z(x) = A\sin\left(\frac{2\pi}{\lambda}x + \frac{\pi}{2}\right)$$

3) In the functions above, if z is a "length" and x is "time", what are the units of A and λ?

4) Write out the first and second derivatives, $\mathrm{d}z/\mathrm{d}x$ and $\mathrm{d}^2z/\mathrm{d}x^2$, of the function in (1). What are the units of these derivatives?

5) Plot the following function $n(t)$ for: (a) $N = 1$, $\tau = 1$; (b) $N = 1$, $\tau = 2$. Plot both cases (a) and (b) on the same graph over $t = 0$ to $t = 4$.

$$n(t) = Ne^{-t/\tau}$$

6) In the function above, if n is a "length" and t is "time", what are the units of N and τ?

7) Write out the first and second derivatives, $\mathrm{d}n/\mathrm{d}t$ and $\mathrm{d}^2n/\mathrm{d}t^2$, of the function in (5). What are the units of these derivatives?

8) Plot the following function $T(x)$ for: $T_0 = 2$, $X = 1$, $\lambda = 2$; $x = 0$ to $x = 4$.

$$T(x) = T_0 e^{-x/X}\cos\left(\frac{2\pi}{\lambda}x\right)$$

9) In the function above, if T is a "temperature" and x is a "length", what are the units of T_0, X and λ?

10) What is the slope, $\mathrm{d}T/\mathrm{d}x$, of the function in (8) at the two positions $x = 0$ and $x = \infty$?

11) Evaluate the following indefinite integrals.

$$\int \cos(\omega x)\,\mathrm{d}x\,, \qquad \int \sin(\omega x)\,\mathrm{d}x$$

12) Evaluate the following definite integrals.

$$\int_0^{\lambda} \cos\left(\frac{2\pi}{\lambda}x\right) \mathrm{d}x \,, \qquad \int_0^{\infty} e^{-t/\tau}\, \mathrm{d}t$$

13) In the integrands above, if x is a "length" and t is "time", what are the units of the integral quantities?

14) Consider the function $f(x)$. Suppose that you know the value of this function $f(a)$ at the specific value $x = a$. You may recall that Taylor's formula (or "series") can be used to estimate the value of $f(x)$ at arbitrary x close to a. Look up Taylor's formula and write it out. (We will use Taylor's formula in class, and discuss a nice geometrical interpretation for what it does and why it works.)

15) Evaluate the following functions.

$$\ln(e^{a}) \,, \qquad \log(10^{b})$$

16) In describing the decay of a radioactive element, physicists might represent this process by:

$$N(t) = N_0 e^{-t/\tau} \qquad \text{or} \qquad \frac{N(t)}{N_0} = e^{-t/\tau}$$

where N is the number of parent atoms at time t, N_0 is the initial number of parent atoms (at $t = 0$), and τ is a characteristic time constant that is specific to the radioactive element. In contrast, isotope geologists might prefer to describe the decay process by:

$$\frac{N(t)}{N_0} = e^{-0.6931\, t/\lambda}$$

where λ is the "half-life" of the radioactive element, namely the time required for one-half of the initial number of parent atoms to decay. Comparing these two expressions, evidently $\lambda = 0.6931\tau$. Show how these expressions are equivalent.