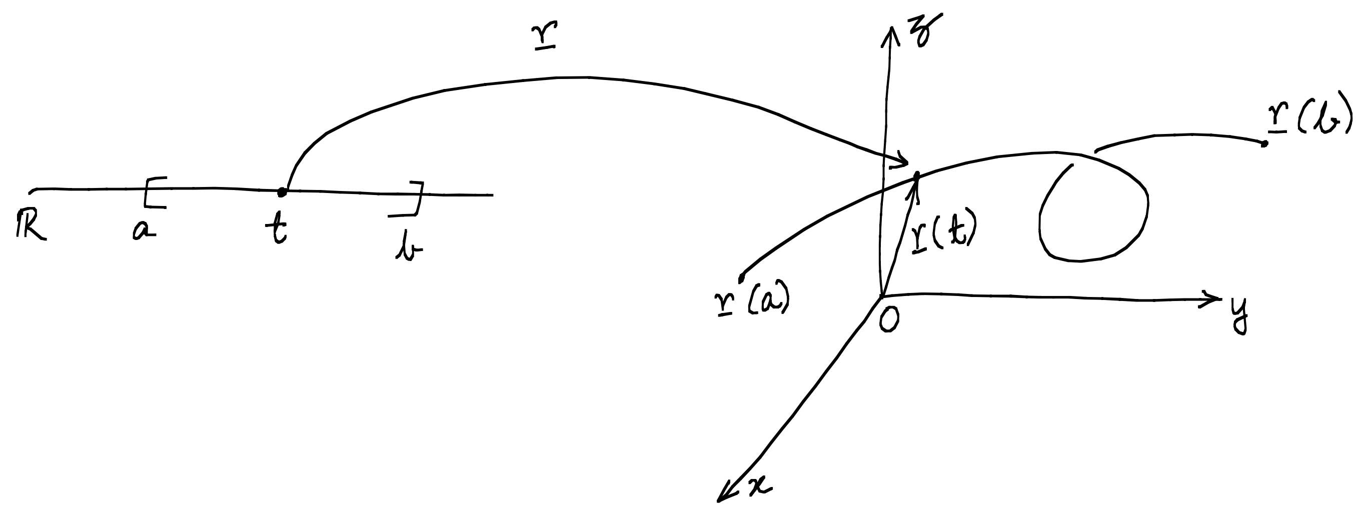
Vector valued functions



If $\underline{r}$ is a vector valued function, then

$\underline{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\underline{i} + g(t)\underline{j} + h(t)\underline{k}$

for some scalar valued functions f, g, h which are also called <u>component function</u> of $\underline{r}$.

<u>Remark</u>: (i) Every vector valued function $\underline{r}$ gives rise to a curve and a direction.

(ii) Two <u>different</u> vector valued functions might give the same curve.

Limits

If $\underline{r} = \langle f, g, h \rangle$, then

$$\lim_{t \to a} \underline{r}(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$$

if the limits of the component function exist.

Derivatives

$$\frac{d}{dt}(\underline{r}(t)) = \underline{r}'(t) \underset{\text{definition}}{=\!=} \lim_{h\to 0} \frac{\underline{r}(t+h) - \underline{r}(t)}{h}$$

if the limit exists.

__Theorem__ If $\underline{r} = \langle f, g, h \rangle$, then $\underline{r}' = \langle f', g', h' \rangle$.

__Theorem__ If $\underline{r}'$ exists at t_0, then the vector $\underline{r}'(t_0)$ is parallel to the tangent line to the curve given by $\underline{r}$ at the point with position vector $\underline{r}(t_0)$.

__Definition:__ __Unit tangent vector__ of $\underline{r}$ is a vector valued function denoted by $\underline{T}$ and is given by $\underline{T}(t) = \dfrac{\underline{r}(t)}{|\underline{r}(t)|}$.

__Rules:__ If c is a fixed scalar, f is a scalar-valued function, and $\underline{u}$ & $\underline{v}$ are vector valued functions, then

(i) $(\underline{u} \pm \underline{v})' = \underline{u}' \pm \underline{v}'$ (ii) $(c\,\underline{u})' = c\,\underline{u}'$

Product Rule:

(iii) $(f\,\underline{u})' = f'\,\underline{u} + f\,\underline{u}'$ (iv) $(\underline{u}\cdot\underline{v})' = \underline{u}'\cdot\underline{v} + \underline{u}\cdot\underline{v}'$

(v) $(\underline{u}\times\underline{v})' = \underline{u}'\times\underline{v} + \underline{u}\times\underline{v}'$

Chain Rule (vi) $(\underline{u}\circ f)' = f'\,(\underline{u}'\circ f)$, that is, $\frac{d}{dt}\left[\underline{u}(f(t))\right] = f'(t)\,\underline{u}'(f(t))$

(vii) If $\underline{u}$ is a constant vector, then $\underline{u}' = \underline{0}$.

Integration

If $\underline{r} = \langle f, g, h \rangle$, then

$$\int_a^b \underline{r}(t)\,dt = \left\langle \int_a^b f(t)\,dt,\ \int_a^b g(t)\,dt,\ \int_a^b h(t)\,dt \right\rangle.$$

Angle between curves

The angle between two curves at a particular point of intersection is given by the angle between the two tangent vectors to the curves at the point of intersection.

Formula If the curves given by $\underline{r}_1$ & $\underline{r}_2$ intersect at t_0 (i.e., $\underline{r}_1(t_0) = \underline{r}_2(t_0)$), then the angle between $\underline{r}_1$ & $\underline{r}_2$ at $\underline{r}_1(t_0) = \underline{r}_2(t_0)$ is given by

$$\cos^{-1}\left(\underline{T}_1(t_0) \cdot T_2(t_0)\right)$$

Where $\underline{T}_1$ & $\underline{T}_2$ are unit tangent vectors of $\underline{r}_1$ & $\underline{r}_2$