

Math 175 Exam 3

Name(print):

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This is a paper-based exam. No calculator is allowed for this exam. To get full points, you must justify your answer(s).

1. (22 points) Evaluate the following integrals.

(a) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} dy dx.$

(b) $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3) dy dx.$

2. (12 points) Rewrite the integral $\int_{-1}^1 \int_x^1 \int_0^{1-y} f(x, y, z) dz dy dx$ as an iterated integrals in the order $dx dy dz$.

3. (24 points, 8 points each) Evaluate the following integrals.

(a) $\int_C zdx + xdy + ydz$, where C is the line segment from $(1, 0, 1)$ to $(2, 3, 1)$.

(b) $\int_C y^3 dx - x^3 dy$, C is the circle $x^2 + y^2 = 9$.

(c) $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = \langle e^x + x^2y, e^y - xy^2 \rangle$, and C is the circle $x^2 + y^2 = 25$ oriented clockwise.

4. (20 points) The joint density function for a pair of random variables X and Y is

$$f(x, y) = \begin{cases} Cxy, & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) find the value of the constant C .

(b) Find $P(X \leq 1, Y \geq 1)$.

(c) Find $P(X + Y \leq 1)$.

5. (10 points) Evaluate the line integral $\int_C \tan y dx + x \sec^2 y dy$, where C is any curve from $(1, 0)$ to $(2, \pi/4)$.

6. (12 points) Let $\vec{f}(x, y) = \frac{(2x^3 + 2xy^2 - 2y)\vec{i} + (2y^3 + 2x^2y + 2x)\vec{j}}{x^2 + y^2}$. Show that $\int_C \vec{F} \cdot d\vec{r}$ is a constant for every positively oriented simple closed path that encloses origin, and find this constant.