

## Math 274 Homework Two

Choose to work out FOUR among the following six problems. Extra credit is available if you work out more than four problems. Due: Tuesday, Jan 29, 2008

- (1) Prove bijectively that the Catalan number  $C_n$  counts the arrangements of  $2n$  people in 2 rows of  $n$  such that heights increases in each row and column.
- (2) A fair coin is flipped  $2n$  times, keeping a running total of heads and tails. Compute the probability that the lead changes, given that they are tied at the end.
- (3) Prove the identity below by counting in two ways.

$$\sum_k \frac{1}{k+1} \binom{2k}{k} \binom{2n-2k}{n-k} = \binom{2n+1}{n}$$

- (4) Place  $2n$  points on a circle. Prove bijectively that the number of ways to pair the points by drawing noncrossing chords equals the number of ballot lists of length  $2n$ . (Page 58, 1.3.19)
- (5) Obtain a recurrence for the number of pairings of  $2n$  people.
- (6) Prove bijectively that

$$\hat{F}_{n+m} = \hat{F}_n \hat{F}_m + \hat{F}_{n-1} \hat{F}_{m-1}.$$

Conclude that for each positive integer  $k$ ,  $\hat{F}_{n-1}$  divides  $\hat{F}_{kn-1}$ .