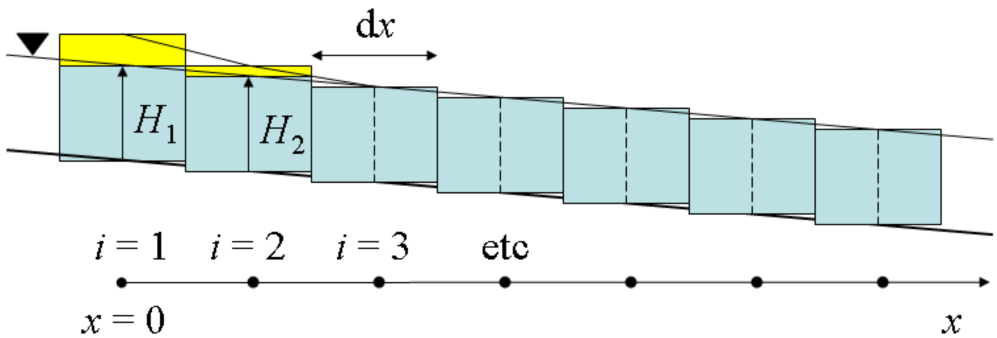
MATLAB Exercise 5: Floodwave Motion

The accompanying M-file consists of a rudimentary "flood routing" algorithm that is commonly used in flood prediction for rivers — small and large. For example, the basic time-stepping "for loop" near the end of the M-file is *the* essential ingredient in routing algorithms used by the National Weather Service to forecast flooding on small- to medium-sized rivers, and by the U.S. Army Corps of Engineers to forecast flooding on the Cumberland, Tennessee, Ohio, Missouri and Mississippi Rivers. The ingredients of the M-file are as follows (look over the algorithm as you read this):

The river is incremented into a series of spatial nodes. Each node is considered to be at the center of a short element (or "reach") of distance dx. Thus,



Notice that the water surface in any element is slightly higher than the water surface in the next downstream element (Figure, blue), although in actuality the water surface varies as a smooth curve. Nonetheless, we will assume that the depth at a node "represents" the average depth of its element. Also note that this declining water surface downstream, with average slope *S*, means steady downstream flow. Using a Manning-like formula, the flow speed *U* is:

$$U = \frac{1}{N} H^{2/3} S^{1/2} \tag{1}$$

Now, the upstream (boundary) node is "handed" a stage hydrograph. That is, the stage (which we will use here to also mean the flow depth *H*) is specified as a function of time at node $i = 1$, independent of the stage at all downstream nodes ($i = 2, 3, 4, ...$). With the rising stage at $i = 1$, the water surface at this first element is higher (than average) than the stage at node $i = 2$ (Figure, yellow). This means that the slope *S* between nodes $i = 1$ and $i = 2$ is greater, so the flow speed *U* from node $i = 1$ to node $i = 2$ increases. In turn, the water surface in the second element must increase (by conservation of mass) to accommodate this upstream flow and, in doing so, the slope between nodes 2 and 3 must increase, thereby increasing flow from node 2 to 3, and so on downstream.

More generally, by conservation of mass in any element,

$$\frac{dV}{dt} = Q_{in} - Q_{out} \tag{2}$$

The element volume $V = WH\mathrm{d}x$, where W is the channel width, and the discharge $Q = WHU$. Substituting and assuming that the width is constant,

$$\frac{dH}{dt} = \frac{H_{in}U_{in} - H_{out}U_{out}}{dx} \tag{3}$$

The next step is to put this into algorithmic form.

Let's denote the stage (water depth) at node i and time t as H_i^t. Looking upstream, the stage of the flow entering the ith element, H_{in}, should be approximated as the average between the stage at node i - 1 and the stage at node at i. So $H_{in} \approx (H_{i-1}^t + H_i^t)/2$. Looking downstream, $H_{out} \approx (H_i^t + H_{i+1}^t)/2$. We treat the "in" and "out" flow speeds in (3), U_{in} and U_{out}, the same way. So, for example,

$$U_{in} = \frac{1}{N} H_{in}^{2/3} S_{in}^{1/2} \tag{4}$$

The numerical approximation of the slope S_{in} follows the development above for the depth H_{in}. Namely, $S_{in} \approx (H_{i-1}^t - H_i^t)/\mathrm{d}x + S_0$, where S_0 is the mean slope of the river ("meanslope" in the M-file). Thus the slope S_{in} is the sum of the overall (constant) average slope of the river plus the slope that is due to differences in stage (water depth) between nodes i and i - 1.

Questions

1) Show the mathematics of getting from Equation (2) to Equation (3) above; provide a clear explanation of your work.

2) Run the M-file as given. The resulting plot shows a floodwave (river stage) that is beginning to propagate downstream. Now, go to line 59 and systematically change the value of the upper limit of the "for loop" (not to exceed the maximum "kmax") to explore how the floodwave changes at it moves downstream. Describe the behavior of the floodwave.

3) Change the roughness coefficient from N = 0.07 to N = 0.1 (rough to rougher). Explain in *physical terms* the difference in floodwave propagation.

4) Estimate the floodwave speed c (from your images) versus the average flow speed U (using (1) above with H = "stagemin" and S = "meanslope"). Show your work and explain any differences between the floodwave speed c and the flow speed U.