

Math 175 Exam 2

Name(print):

Problem 1:	/8
Problem 2:	/12
Problem 3:	/14
Problem 4:	/10
Problem 5:	/10
Problem 6:	/12
Problem 7:	/10
Problem 8:	/12
Problem 9:	/12
Total:	/100

This is a paper-based exam. No calculator is allowed for this exam. To get full points, you must justify your answer(s).

1. (8 points) Find and sketch the domain of the function

$$f(x, y) = \sqrt{x^2 + y^2 - 1} + \ln(4 - x^2 - y^2).$$

2. (12 points) Find the tangential and normal components of the acceleration vector, given the position function

$$\vec{r}(t) = t\vec{i} + \cos^2 t\vec{j} + \sin^2 t\vec{k}.$$

3. (14 points) Find the limits, if it exists, or show that the limit does not exist.

(a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2x^2y}{x^4 + y^2}$$

(b)

$$\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln(x^2 + y^2)$$

(hint: use polar coordinates for (x, y) .)

4. (10 points) Find the linear approximation of the function $f(x, y) = \sqrt{x + e^{4y}}$ at point $(3, 0)$ and use it to approximate the number $\sqrt{3.01 + e^{-0.04}}$.

5. (10 points) If $z=f(x,y)$, where $x = s + t, y = s - t$, show that

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \frac{\partial z}{\partial s} \frac{\partial z}{\partial t}.$$

6. (12 points) Find the directional derivative of the function at the given point in the direction of the vector \vec{v} . At which direction is the rate of change of f at the given point maximized?

$$f(x, y, z) = \frac{x}{y + z}, (4, 1, 1), \vec{v} = \langle 1, 2, 3 \rangle.$$

7. (10 points) Show that every plane that is tangent to the cone $x^2 + y^2 = z^2$ passes through the origin.

8. (12 points) Find the local maximum and minimum values and saddle points of the function $f(x, y) = (1 + xy)(x + y)$.

9. (12 points) Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint:

$$f(x, y) = x^2y, x^2 + 2y^2 \leq 6.$$