Math 274 Homework Five

Choose to work out at least six problems from the following nine problems, including the first one. Due: Tuesday, Feb 26, 2008

(1) Let a_n be the number of nonnegative integer solutions to $4e_1+2e_2+e_3+2e_4 = n$. Find the generating function for $\langle a\rangle$ and find a_n.

(2) In how many ways can one pick 25 coins that are pennies, nickels, or dimes, with at least three nickels, at most five dimes and an even number of pennies?

(3) Let $b_{n,k}$ be the number of k-subsets of $[n]$ having no consecutive integers. Let $B_k(x) = \sum_{n\geq 0} b_{n,k}x^n$. Determine $B_k(n)$. (hint: $b_{n,k}$ is the number of integer solutions to $\sum_{i=0}^{k} e_i = n$ with certain constrains.)

(4) Let a_n be the number of ways to select $r \in N$, roll a six-sided die r times, and obtain a sum of n. express the generating function for $\langle a\rangle$ as a ratio of tow polynomials.

(5) Let $c(n, k)$ be the number of permutations of $[n]$ with k cycles. prove bijectively that $c(n + 1, m + 1) = \sum_{k=m}^{n} c(n, k)\binom{k}{m}$. (hint: use the idea of the cycle representation of permutations.)

Use generating function to evaluate the sums below:

(6)
$$\sum_{k=0}^{r}(-1)^k\binom{n}{k}\binom{n}{r-k}.$$

(7)
$$\sum_{j=0}^{k}\binom{n+k-j-1}{k-j}\binom{m+j-1}{j}.$$

(8)
$$\sum_{k}\binom{n+k}{2k}2^{n-k} \text{ for all } n \in N$$

(9)
$$\sum_{k=0}^{n}(-1)^k\binom{n}{k}\binom{m+n-k}{r-k}.$$