

Math 274 Homework Three

Choose to work out at least four problems from the following five problems, including the first one. Due: Tuesday, Feb 5, 2008

- (1) Prove recursively that in each below, a_n is the n^{th} Catalan number.
 - (a) Let a_n be the number of non-crossing pairings of $2n$ points on a circle.
 - (b) let a_n be the number of configurations of pennies on a base row of n pennies, where pennies can be added so that each penny not in the base rests on two in the row immediately below it.

- (2) Let $b_{n,k}$ be the number of k -element subsets of $[n]$ containing no two consecutive integers. Obtain a recurrence relation in two indices for these numbers.

- (3) Let h_n equal the number of different ways in which the squares of a 1-by- n chessboard can be colored, using the colors red, white and blue so that no two squares that are colored red are adjacent. Find and verify a recurrence relation that h_n satisfies. Then find a formula for h_n .

- (4) Solve the following recurrence relation.
$$a_n = 5a_{n-1} - 6a_{n-2} + 2^n.$$

- (5) Solve the following recurrence relation.
$$h_n = 5h_{n-1} - 6h_{n-2} - 4h_{n-3} + 8h_{n-4}, \quad (n \geq 4), \quad \text{with } h_0 = 0, h_1 = 1, h_2 = 1, h_3 = 2.$$