# Limits & Continuity

**Definition:** Let $f$ be a function of 2 variables whose domain contains a neighbourhood of $(a,b)$. We say that limit of $f(x,y)$ exists & equals to $L$ as $(x,y)$ approaches $(a,b)$ if:

for every $\varepsilon > 0$, there exists $\delta > 0$ such that

$|f(x,y) - L| < \varepsilon$ whenever $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$

**Fact 1:** (Useful for showing non-existence of a limit)

If $f(x,y) \to L_1$ as $(x,y) \to (a,b)$ along a path $C_1$ & $f(x,y) \to L_2$ as $(x,y) \to (a,b)$ along a path $C_2$ where $C_1$ & $C_2$ both approach $(a,b)$ & $L_1 \neq L_2$, then $\lim_{(x,y)\to(a,b)} f(x,y)$ does not exist.

**Fact 2:** (Useful to show existence of a limit)

If $g(x,y) \leq f(x,y) \leq h(x,y)$ for all $(x,y)$ in a neighbourhood of $(a,b)$ & $\lim_{(x,y)\to(a,b)} g(x,y) = L = \lim_{(x,y)\to(a,b)} h(x,y)$, then $\lim_{(x,y)\to(a,b)} f(x,y) = L$

**Definition:** A function $f$ of 2 variables is said to be continuous at $(a,b)$ if $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

**Remark:** All of the above makes sense for 3 variables with appropriate changes, namely, replacing $(x, y)$ by $(x, y, z)$, $(a, b)$ by $(a, b, c)$, etc.

**Fact 3:** Any polynomial, rational, algebraic, trigonometric, inverse-trigonometric, exponential, logarithmic hyperbolic & inverse-hyperbolic functions of 2 or 3 variables are continuous in their domain. In other words, limit of these type of functions at any point in their domain can be found by direct substitution.