EXAM #3
PHYSICS 117a
Fall 2007

Only calculators and pens/pencils are allowed on your desk. No cell phones or additional scrap paper. You have 2.0 hours to complete the exam.

Name ______________________________

Please circle which section you are in:

Section 1, Professor Ernst, MWF 10:10-11:00
Section 2, Professor Sheldon, TTh 11:00=12:15

I pledge my honor that I have neither given nor received aid on this work.

Signature ______________________________

MC(50) ____ Prob11 _____ Prob12 ____ Prob13 ____ Total ____

Multiple choice, circle the correct answer (5 points each).

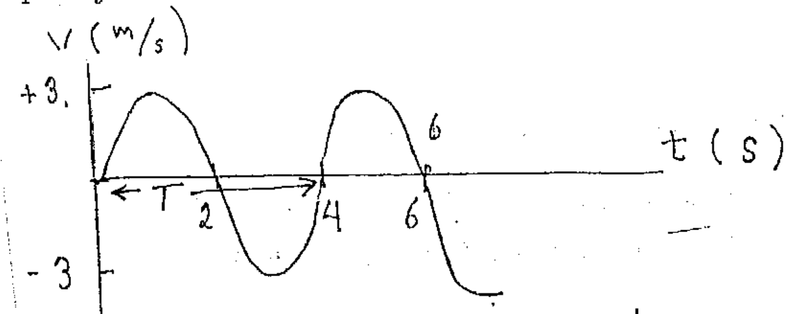
1.) If the frequency of an harmonic oscillator doubles, by what factor does the maximum value of the acceleration change?

A. $\sqrt{2}$
B. 4
C. 2
D. 2π
E. 4π

$y(x,t) = A \cos(kx - \omega t)$
$v_y(x,t) = dy/dt = A\omega \sin(kx - \omega t)$
$a_y(x,t) = dv/dt = A\omega^2 \cos(kx - \omega t)$
$\omega \to 2\omega, \quad a_{max} = A\omega^2 \to 4\, a_{max}$

2.) A mass of 6 kg is attached to a spring and oscillates horizontally on a frictionless surface. The velocity of the mass as a function of time is shown in the graph below. What is the angular frequency ω of the oscillations?

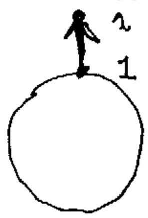
A. 0.67 s^{-1}
B. 1.00 s^{-1}
C. 1.57 s^{-1}
D. 3.14 s^{-1}
E. 6.28 s^{-1}



$T = 4\,s, \quad f = \frac{1}{T} = 0.25\,s^{-1}, \quad \omega = 2\pi f = 1.57\,s^{-1}$

3.) Superman throws a rock into space. He throws the rock vertically upward from the north pole and it goes up a height above the ground of two Earth radii before falling back. What was the velocity of the rock when it left Superman's hand? You may neglect air friction.

A. 7.91×10^3 m/s
B. 9.14×10^3 m/s
C. 1.18×10^4 m/s
D. 1.58×10^4 m/s
E. 1.88×10^4 m/s



Conserve energy between point 1 and 2. Note there is potential energy at 1, the surface of the Earth

$E_1 = E_2$

$K_1 + U_1 = U_2 \quad (v = 0 \text{ at top})$

$\frac{1}{2} m v^2 - \frac{GMm}{r_1} = - \frac{GMm}{r_2}$

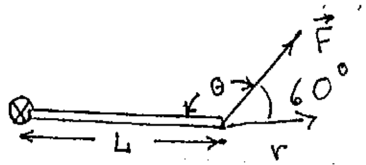
radius of Earth from table.

$v^2 = 2\left[- \frac{6.67 \times 10^{-11}\, 5.97 \times 10^{24}}{6.38 \times 10^{6}} + \frac{6.67 \times 10^{-11}\, 5.97 \times 10^{24}}{3 \times 6.38 \times 10^{6}} \right]$

note

$v = 9.14 \times 10^{3} \; m/s$

4.) A stick of length $L = 0.75$ lies along the x-axis. The left end is attached to an axle. A force of 2.5 N is applied to the right end at an angle of $\theta = 120°$ as drawn. The torque on the stick about the axle, $\vec{\tau}$, is



$\tau = F r \sin\theta$ $\angle$ angle between $\vec{r}$ + $\vec{F} \rightarrow 60°$

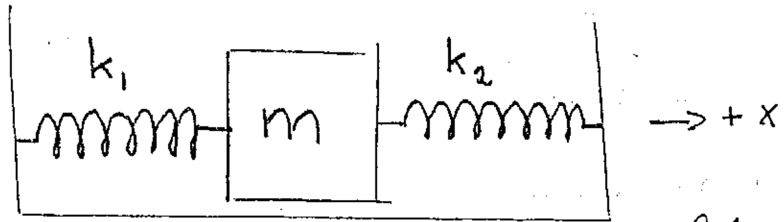
A. 0.94 Nm into the paper
B. 0.94 Nm out of the paper
C. 1.6 Nm into the paper
D. 1.6 Nm out of the paper
E. 1.9 Nm toward the right
F. 1.9 Nm toward the left

$\tau = 2.5\,N \times 0.75\,m \times \sin 60°$

$= 1.6\,Nm$

right hand rule $\rightarrow$ out of page

5.) A 2 kg block on a frictionless table is connected to two springs with spring constants k_1 and k_2 that are attached to the opposite sides of the block and whose opposite ends are attached to walls, as shown in the figure. What is the angular frequency of oscillation if the spring constants are $k_1 = 2.2$ N/m and $k_2 = 5.0$ n/M?



A. 0.30 rad/s
B. 0.42 rad/s
C. 1.9 rad/s
D. 2.6 rad/s
E. 4.2 rad/s

Move mass a little (Δx) to right, $F_1 = -k_1 \Delta x$
and $F_2 = -k_2 \Delta x$ {minus means left}
$F_{tot} = -(k_1 + k_2)\Delta x$, $\omega = \sqrt{\frac{k_1+k_2}{m}} = 1.9$

6.) A potter's wheel, with a moment of inertia of 39 kg m^2, is spinning freely at 40 rpm (revolutions/minute). The potter drops a lump of clay onto the wheel and it sticks at a distance of 1.2 m from the axis. If the subsequent angular speed of the wheel and the clay system is 32 rpm, what is the mass of the clay?

Conserve angular momentum

A. 5.9 kg
B. 6.8 kg
C. 7.5 kg
D. 8.3 kg

$I_i = 39\ kg\,m^2$ — I for clay
$I_f = I_i + m r^2$
$= 39\ kg\,m^2 + m\,(1.2\,m)^2$

$I_i \omega_i = I_f \omega_f$
$39\ kg \cdot m^2 \times 40\,rpm = (39\ kg\,m^2 + m \times 1.44\,m^2)\, 32\,rpm$

$\frac{39\ kg \cdot m^2 \times 40}{32} - 39\ kg\,m^2 = 1.44\,m^2 \times m$, $m = 6.8\ kg$

7.) A 0.25 kg mass undergoes harmonic oscillation with a total energy of 8.7 J. If the mass oscillates between a distance of 30 cm from the wall to a distance of 70 cm from the wall, what is the oscillation frequency?

A. 2.0 cycles/s
B. 3.3 cycles/s
C. 4.6 cycles/s
D. 6.6 cycles/s
E. 8.2 cycles/s

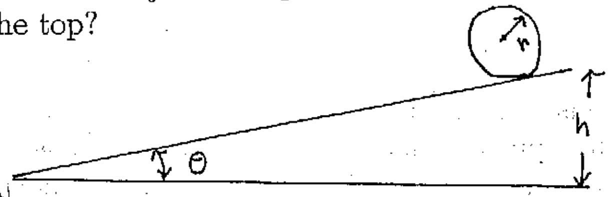
$A = \frac{70\,cm - 30\,cm}{2} = 20\,cm = 0.20\,m$ (2 ← note)

When spring is maximally stretched, energy is all potential ($v = 0$)

$E = \frac{1}{2} k A^2$, $k = \frac{2E}{A^2} = \frac{2 \times 8.7 \frac{kg\,m}{s^2}}{(.2\,m)^2} = 435 \frac{N}{m}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{435\ N/m}{.25\ kg}} = 4.7\ s^{-1}$, $f = \frac{\omega}{2\pi} = 6.6 \frac{cycle}{s}$

8.) A solid uniform sphere of mass 2.0 kg and radius 1.7 m rolls down an incline plane of height 4.3 m and which makes an angle of 12° with the horizontal. What is the angular velocity of the sphere at the bottom of the incline plane if it starts from rest at the top?



A. 4.6 rad/s
B. 5.5 rad/s
C. 7.8 rad/s
D. 8.7 rad/s
E. 9.8 rad/s

Conserve energy

$U = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$v = \omega r$, $I = \frac{2}{5} m r^2 \Rightarrow$

$\cancel{m} g h = \frac{1}{2} m r^2 \omega^2 + \frac{1}{5} m r^2 \omega^2$

$= (\frac{7}{10} \cancel{m} r^2) \omega^2$

$9.8 \frac{m}{s^2} \times 4.3\,m = 0.7 (1.7\,m)^2 \omega^2$

$\omega = 4.6\ s^{-1}$

9.) Air passes over the top of the wing of an airplane at 35 m/s while passing under the bottom of the wing at 32 m/s. If the area of the wing 24 m^2, what is the net upward force on the wing? The density of the air is 1.2 kg/m^3.

A. 0.16×10^3 N
B. 0.32×10^3 N
C. 0.76×10^3 N
D. 1.4×10^3 N
E. 2.9×10^3 N

Apply Bernoulli to top and bottom of wing

$P_t + \frac{1}{2} \rho v_t^2 = P_b + \frac{1}{2} \rho v_b^2$

$P_b - P_t = \Delta P = \frac{1}{2} \rho v_t^2 - \frac{1}{2} \rho v_b^2$

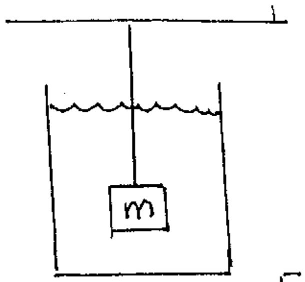
$= \frac{1}{2} \rho (v_t^2 - v_b^2) = \frac{1}{2}\, 1.2\, kg/m^2 \times [(35 \frac{m}{s})^2 - (32 \frac{m}{s})^2]$

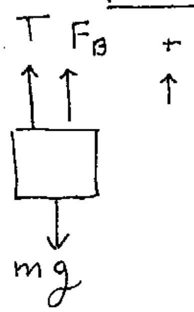
$\Delta P = 121 \frac{N}{m^2}$

$F_{net} = \Delta P \cdot A = 2.9 \times 10^3 N$

10.) A cubic metal block of density 3.2×10^4 kg/m^3 and sides of length 0.75 m is suspended by a rope while submerged in a liquid with density 1.0×10^4 kg/m^3. What is the tension in the rope?

A. 1.6×10^4 N
B. 3.2×10^4 N
C. 6.1×10^4 N
D. 7.3×10^4 N
E. 9.1×10^4 N





$$F_{net} = 0$$

$$T + F_B - mg = 0$$

$$F_B = \rho_{liq} V g$$

$$mg = (\rho_m V) g$$

$$T = mg - F_B$$

$$= \rho_m V g - \rho_{liq} V g$$

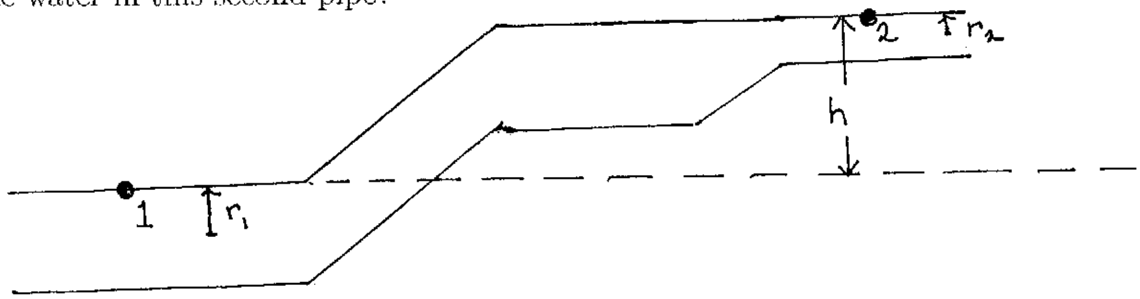
$$= (\rho_m - \rho_{liq}) V g$$

$$= \left(3.2 \times 10^4 \frac{kg}{m^3} - 1.0 \frac{kg}{m^3}\right) \times (0.75 m)^3 \times 9.8 \, m/s^2$$

$$= 9.1 \times 10^4 \text{ N}$$

Problem, show your work clearly and logically, partial credit will be given. Please draw a box around your answer.

11.) [16 pointa] A liquid, density 1.00×10^4 kg/m^3, is flowing with a cylindrical pipe of radius 0.40 m with a velocity of 8.5 m/s. The gauge pressure of the water in this pipe is measured to be 8.3×10^6 N/m^2. The pipe goes uphill to a height of 12 m and is attached smoothly to a pipe of radius 0.20 m. What gauge pressure is measured for the water in this second pipe?



$V_1 = 8.5$ m/s

Use $A_1 V_1 = A_2 V_2$ to get V_2

$$\pi r_1^2 V_1 = \pi r_2^2 V_2$$

$$V_2 = \frac{r_1^2}{r_2^2} V_1 = \frac{r_1^2}{(\frac{1}{2} r_1)^2} V_1 = 4 V_1 = 34 \text{ m/s}$$

Apply Bernoulli to points 1 and 2

$$P_1 + \frac{1}{2}\rho V_1^2 + \cancel{\rho g h_1} = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

0

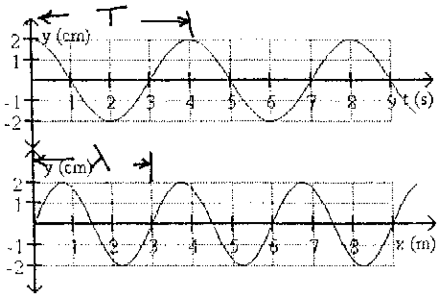
$$P_2 = P_1 + \frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho V_2^2 - \rho g h_2$$

$$= 8.3 \times 10^6 \text{ N/m}^2 + \frac{1}{2} 10^4 (8.5)^2 \frac{N}{m^2} - \frac{1}{2}(10^4)(34)^2 \frac{N}{m^2} - 10^4 (9.8) 12 \frac{N}{m^2}$$

$$\boxed{P_2 = 1.7 \times 10^6 \frac{N}{m^2}}$$

Problem, show your work clearly and logically, partial credit will be given. Please draw a box around your answer.

12.) [17 points] Below are graphs of a wave on a string. The first is the wave as a function of time measured at a fixed x and the second is the wave as a function of distance measured at a fixed time t. If the linear density of the string is 2.3×10^{-3} kg/m, what is the tension T in the string?



$T = 4\,s$

$\lambda = 3\,m$

$$v = \lambda f = \frac{\lambda}{T} = \frac{3m}{4s} = 0.75\ m/s$$

and

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow T = v^2 \mu = (0.75\ m/s)^2 \times 2.3 \times 10^{-3}\ kg/m$$

$$= 1.3 \times 10^{-3}\ \frac{kg\,m}{s^2}$$

$$\boxed{T = 1.3 \times 10^{-3}\ N}$$

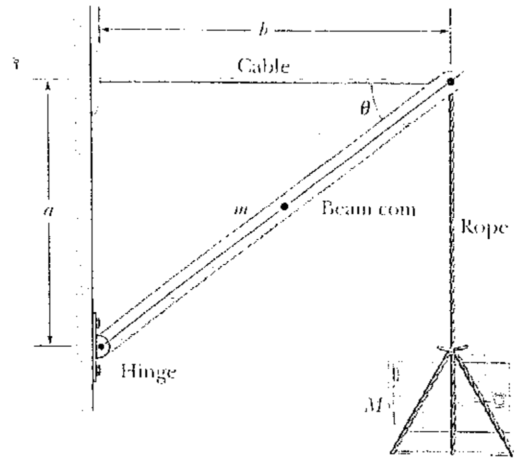
Problem, show your work clearly and logically, partial credit will be given. Please draw a box around your answer.

13.) [17 points] A safe, of mass $M = 430$ kg, is hanging by a rope from a uniform beam with dimensions $a = 1.9$ m and $b = 2.5$ m. A horizontal cable is attached to the beam at its upper end and to the wall. The mass of the bem is 100 kg.

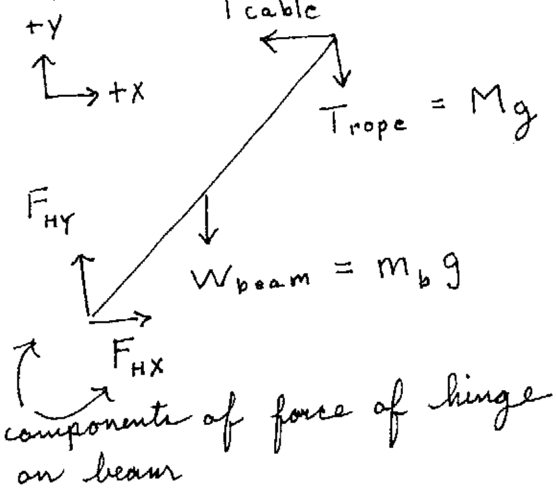
a.) Draw the free body digram showing all the forces acting on the beam.

b.) What is the tension in the horizontal cable?

c.) What is the force of the hinge on the beam, $\vec{F}_{hinge}$?



a.)



b.) Take $\sum \tau_i = 0$, choose $\circlearrowleft +$ {counterclock wise positive}

$\tau = r F \sin\theta = r_\perp F = r F_\perp$

use this as we are given $r_\perp$

Choose hinge as our axis (this will eliminate F_{HX} and F_{HY} from torque equation.)

Note: Rope, $r_\perp = b = 2.5\,m$

Beam, $r_\perp = b/2 = 1.25\,m$

Cable, $r_\perp = a = 1.9\,m$

$$\sum \tau_i = +\underset{\circlearrowleft}{\tau_{cable}} - \underset{\circlearrowright}{\tau_{rope}} - \underset{\circlearrowright}{\tau_{beam}} = 0$$

$$T_{cable}\, a - Mg\, b - m_b g \left(\frac{b}{2}\right) = 0$$

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$$T_{cable} = \frac{Mgb + m_b g \frac{b}{2}}{a}$$

$$= \frac{430\,kg\ 9.8\frac{m}{s^2} \times 2.5m + 100\,kg\ 9.8\frac{m}{s^2} \times 1.25m}{1.9m}$$

$$\boxed{T_{cable} = 6.2 \times 10^3\,N}$$

c.) Use

$$\sum F_x\text{'s} = 0, \quad \rightarrow +$$

$$F_{HX} - T_{cable} = 0$$

$$\boxed{F_{HX} = T_{cable} = 6.2 \times 10^3\,N}$$

$$\sum F_y\text{'s} = 0, \quad \uparrow +$$

$$F_{HY} - Mg - m_b g = 0$$

$$F_{HY} = Mg + m_b g = 430\,kg\ 9.8\frac{m}{s^2} + 100\,kg\ 9.8\frac{m}{s^2}$$

$$\boxed{F_{HY} = 5.2 \times 10^3\,N}$$