

Math 274 Homework Four

Choose to work out at least four problems from the following five problems, including the fifth one. Due: Tuesday, Feb 13, 2008

- (1) Let $b_{n,k}$ be the number of k -element subsets of $[n]$ containing no two consecutive integers. Obtain a recurrence relation in two indices for these numbers.

- (2) Using generating function method to solve the following recurrence relation.
$$a_n = 3a_{n-1} - 2a_{n-2} + 2^n + n \text{ with } a_0 = a_1 = 1.$$

- (3) Let $a_{n,k}$ denote the number of ways to partition n people in a row into k groups so that no two consecutive people are in the same group.
 - (a) Obtain a recurrence relation for $a_{n,k}$.
 - (b) Let $A_k(x) = \sum_{n=0}^{\infty} a_{n,k}x^n$. Use the generating function method to express $A_k(x)$ as a ratio of polynomials.

- (4) Give the exact solution formula (for all n) for the recurrence below:
$$a_n = a_{\lfloor n/2 \rfloor} + 1 \text{ for } n \geq 2, \text{ with } a_1 = 0.$$

- (5) Solve the recurrence $a_n = 2a_{n-1}^2$ with $a_0 = 1$