

Math 274 Homework eight

Choose to work out at least four problems from the following five problems, including the first one. Due: Tuesday, March 25, 2008

- (1) Evaluate the sum below using generating functions, and then give inclusion-exclusion proofs of the resulting identity.

$$\sum_{k=0}^n (-1)^k \binom{n}{k} \binom{m-k}{r}.$$

- (2) Evaluate the sum below, and use the inclusion-exclusion to prove the resulting identity. (hint: for the inclusion-exclusion part, count a particular family of 0, 1-lists that have no consecutive 1s.)

$$\sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \binom{n-k}{r-k}.$$

- (3) Give a combinatorial proof (via inclusion-exclusion) for the identity below.

$$\sum_{k=0}^n (-1)^k \binom{n}{k} \binom{2n-2k}{n-1} = 0.$$

- (4) How many ways are there to arrange $n \geq 3$ differently colored beads in a necklace?
- (5) How many nonequivalent ways are there to color the corners of a regular 6-gon with the colors red and blue, such that three corners are red and four corners are blue?