

$$x = x_0 + \int_{t_0}^t v_x dt \quad v_x = \frac{dx}{dt} \quad v_{x,avg} = \frac{\Delta x}{\Delta t} \quad \theta = \theta_0 + \int_{t_0}^t \omega dt \quad \omega = \frac{d\theta}{dt} \quad \omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

$$v_x = v_{x0} + \int_{t_0}^t a_x dt \quad a_x = \frac{dv_x}{dt} \quad a_{x,avg} = \frac{\Delta v_x}{\Delta t} \quad \omega = \omega_0 + \int_{t_0}^t \alpha dt \quad \alpha = \frac{d\omega}{dt} \quad \alpha_{avg} = \frac{\Delta\omega}{\Delta t}$$

$$v_x = v_{x0} + a_x \Delta t$$

$$x = x_0 + v_{x0} \Delta t + \frac{1}{2} a_x (\Delta t)^2$$

$$v_x^2 = v_{x0}^2 + 2a_x \Delta x$$

Similar equations apply for motion along y and z.

$$\omega = \omega_0 + \alpha \Delta t$$

$$\theta = \theta_0 + \omega_0 \Delta t + \frac{1}{2} \alpha (\Delta t)^2$$

$$\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$$

$$a_{rad} = v^2 / R = \omega^2 R$$

$$a_{tan} = \alpha R$$

$$v_{tan} = \omega R$$

$$s = \theta R$$

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{\tau}_{net} = I\vec{\alpha}$$

$$w = mg \text{ OR } F_g = Gm_1 m_2 / r^2 \quad \vec{F}_{spring} = -k(\Delta\vec{x})$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$f_s \leq \mu_s n \quad f_k = \mu_k n \quad f_r = \mu_r n$$

$$\text{where } |\vec{\tau}| = I\alpha = rF \sin\phi = rF_{tan}$$

$$f_{drag} = kv \text{ (low speed) OR } f_{drag} = Dv^2 \text{ (high speed)}$$

$$I = \sum_i m_i r_i^2$$

$$\Delta K = W_{total}$$

$$W_c = -\Delta U$$

$$W_{NC} = -\Delta U_{int} = \Delta E \text{ where } E = K + U$$

$$\Delta K + \Delta U + \Delta U_{int} = 0$$

$$K_i + U_i + W_{NC} = K_f + U_f$$

$$K_{trans} = \frac{1}{2} mv^2$$

$$K_{rot} = \frac{1}{2} I\omega^2$$

$$W = \int_{s_i}^{s_f} \vec{F} \cdot d\vec{s} \quad P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

$$W = \int_{\theta_i}^{\theta_f} \tau_z d\theta \quad P = \frac{dW}{dt} = \tau_z \omega_z$$

$$U_g = mgy \text{ OR } U_g = -Gm_1 m_2 / r \quad U_{elastic} = \frac{1}{2} k(\Delta x)^2$$

$$F_x(x) = \frac{-dU(x)}{dx}$$

$$\vec{p} = m\vec{v}$$

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = I\vec{\omega}$$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \quad \vec{J} = \int_{t_i}^{t_f} \vec{F}_{net} dt = \Delta\vec{p}$$

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

$I\omega$ applies only for a rigid body around an axis of symmetry

$$p = p_0 + \rho gh$$

$$F_B = m_{fl,disp} g = \rho_{fl} V_{fl,disp} g$$

$$\frac{dv}{dt} = Av = \text{constant}$$

$$p + \rho gy + \frac{1}{2} \rho v^2 = \text{constant}$$

Simple harmonic oscillator: $x(t) = A \cos(\omega t + \phi_0)$

$$\omega = 2\pi f = 2\pi / T \quad \text{where } \omega = \sqrt{k/m} \text{ (spring-mass)}$$

$$\text{OR } \omega = \sqrt{K/I} \text{ (torsion spring)}$$

$$\text{OR } \omega = \sqrt{g/L} \text{ (simple pendulum)}$$

$$\text{OR } \omega = \sqrt{mgd/I} \text{ (physical pendulum)}$$

Slightly damped oscillator: $x(t) = Ae^{-(0.5b/m)t} \cos(\omega' t + \phi_0)$

$$\text{where } \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$y(x,t) = A \cos\left[2\pi\left(\frac{x \mp vt}{\lambda}\right)\right] = A \cos\left[2\pi\left(\frac{x}{\lambda} \mp \frac{t}{T}\right)\right] = A \cos(kx \mp \omega t) \quad \text{where } k = 2\pi / \lambda \quad \text{and} \quad v = \lambda / T = \lambda f$$

transverse, string/spring:

$$v = \sqrt{T/\mu}$$

$$P_{avg} = \frac{1}{2} \omega^2 A^2 \sqrt{\mu T}$$

longitudinal, fluid or bulk solid:

$$v = \sqrt{B/\rho}$$

$$I = \frac{1}{2} \omega^2 A^2 \sqrt{\rho B} = p_{max}^2 / (2\rho v) \quad \text{where } p_{max} = 2\pi BA / \lambda$$

$$I_1 / I_2 = r_2^2 / r_1^2$$

$$\beta = (10 \text{ dB}) \log(I / I_0)$$

$y(x,t) = A_{sw} \sin(kx + \phi_0) \sin(\omega t)$ where $A_{sw} = 2A$ (standing wave)

$$f_n = n \frac{v}{2L} = n f_1$$

$$\lambda_n = 2L / n \quad \text{where } n = 1, 2, 3, \dots \text{ (string or open pipe)}$$

$$f_n = n \frac{v}{4L} = n f_1$$

$$\lambda_n = 4L / n \quad \text{where } n = 1, 3, 5, \dots \text{ (stopped pipe)}$$

Useful Constants

$$g = 9.8 \text{ m/s}^2$$

$$G = 6.673 \times 10^{-11} \text{ N m}^2 / \text{kg}^2$$

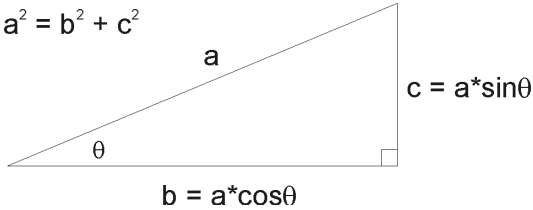
$$I_0 = 1.0 \times 10^{-12} \text{ W / m}^2$$

$$p_{\text{atm}} = 1.013 \times 10^5 \text{ Pa} = 1 \text{ atm}$$

Algebra and trigonometry

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a^2 = b^2 + c^2$$



Astronomical Data

	<u>Mass (kg)</u>	<u>Radius (m)</u>	<u>Orbit radius (m)</u>	<u>Orbit period</u>
Sun	1.99×10^{30}	6.96×10^8	-----	-----
Moon	7.35×10^{22}	1.74×10^6	3.84×10^8	27.3 days
Mercury	3.30×10^{23}	2.44×10^6	5.79×10^{10}	88.0 days
Venus	4.87×10^{24}	6.05×10^6	1.08×10^{11}	224.7 days
Earth	5.97×10^{24}	6.38×10^6	1.50×10^{11}	365.3 days
Mars	6.42×10^{23}	3.40×10^6	2.28×10^{11}	687.0 days
Jupiter	1.90×10^{27}	6.91×10^7	7.78×10^{11}	11.86 yrs
Saturn	5.68×10^{26}	6.03×10^7	1.43×10^{12}	29.45 yrs
Uranus	8.68×10^{25}	2.56×10^7	2.87×10^{12}	84.02 yrs
Neptune	1.02×10^{26}	2.48×10^7	4.50×10^{12}	164.8 yrs
Pluto	1.31×10^{22}	1.15×10^6	5.91×10^{12}	247.9 yrs

*** For each body, "radius" is its radius at the equator and "orbit radius" is its average distance from the sun (for the planets) or from the earth (for the moon).

Moments of Inertia

slender rod (length L), through center

$$I = \frac{1}{12} ML^2$$

slender rod (length L), through end

$$I = \frac{1}{3} ML^2$$

rectangular plate (a by b), through center

$$I = \frac{1}{12} M(a^2 + b^2)$$

rectangular plate (a by b), along edge b

$$I = \frac{1}{3} Ma^2$$

hollow cylinder (inner radius R_1 , outer R_2)

$$I = \frac{1}{2} M(R_1^2 + R_2^2)$$

solid cylinder, (radius R)

$$I = \frac{1}{2} MR^2$$

thin-walled hollow cylinder, (radius R)

$$I = MR^2$$

solid sphere (radius R)

$$I = \frac{2}{5} MR^2$$

thin-walled hollow sphere (radius R)

$$I = \frac{2}{3} MR^2$$